Getting *free* Bits Back from Rotational Symmetries in LLMs

Jiajun He, Gergely Flamich, José Miguel Hernández-Lobato

University of Cambridge

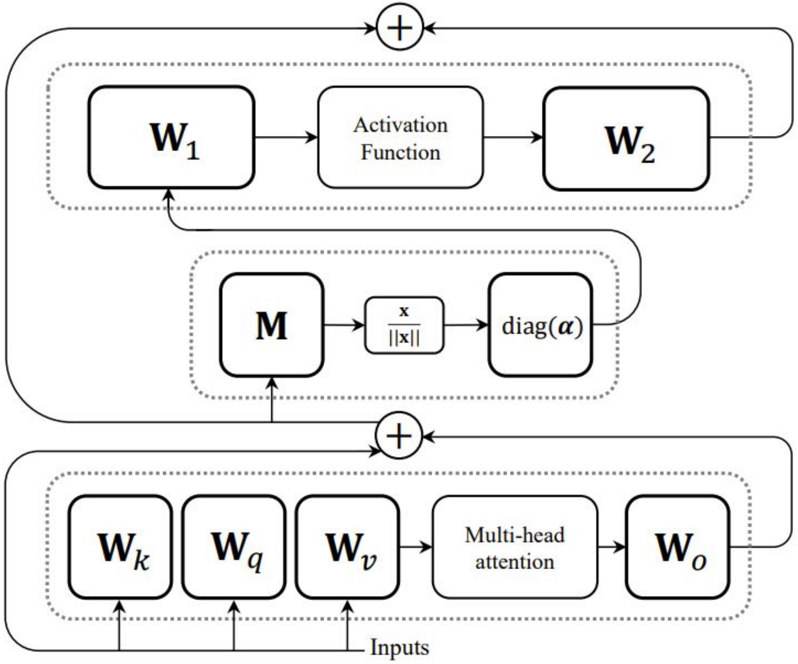
After pruning LLMs,

we can further save 3-5% additional bits *for free*

in storage and transmission

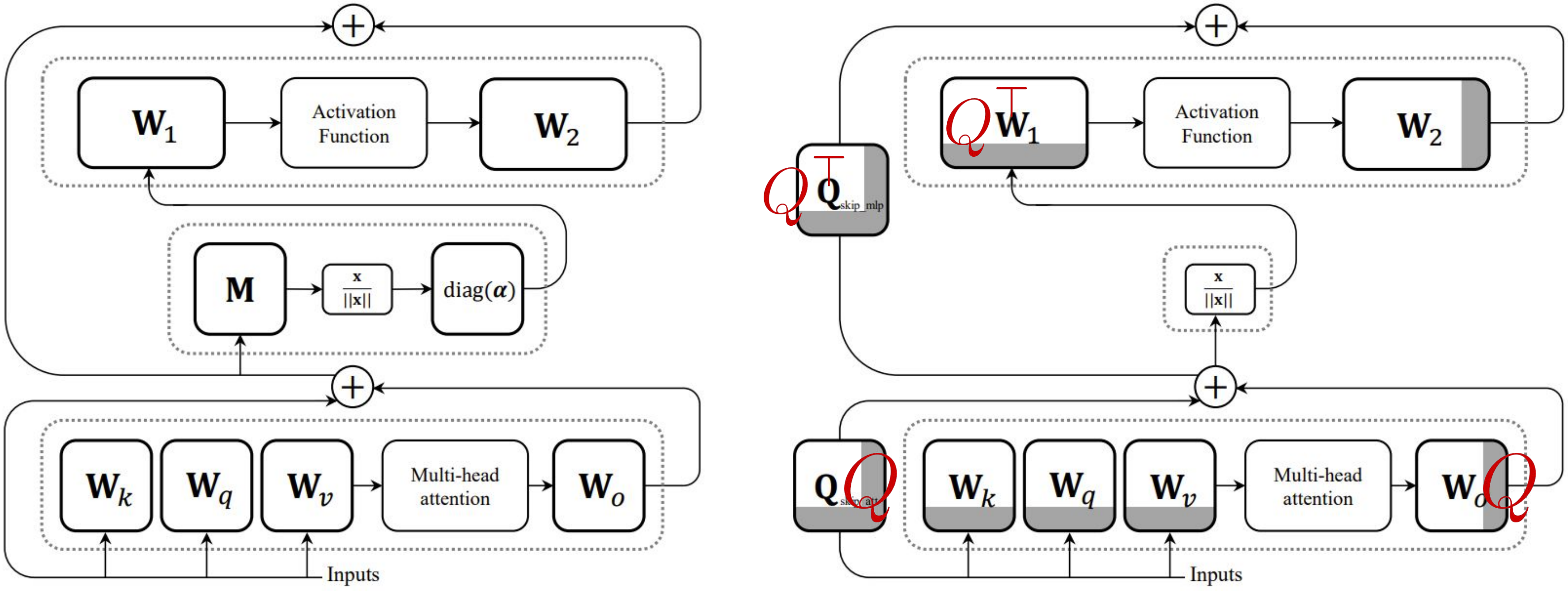
SliceGPT and bits-back coding

SliceGPT



(a) A standard transformer block.

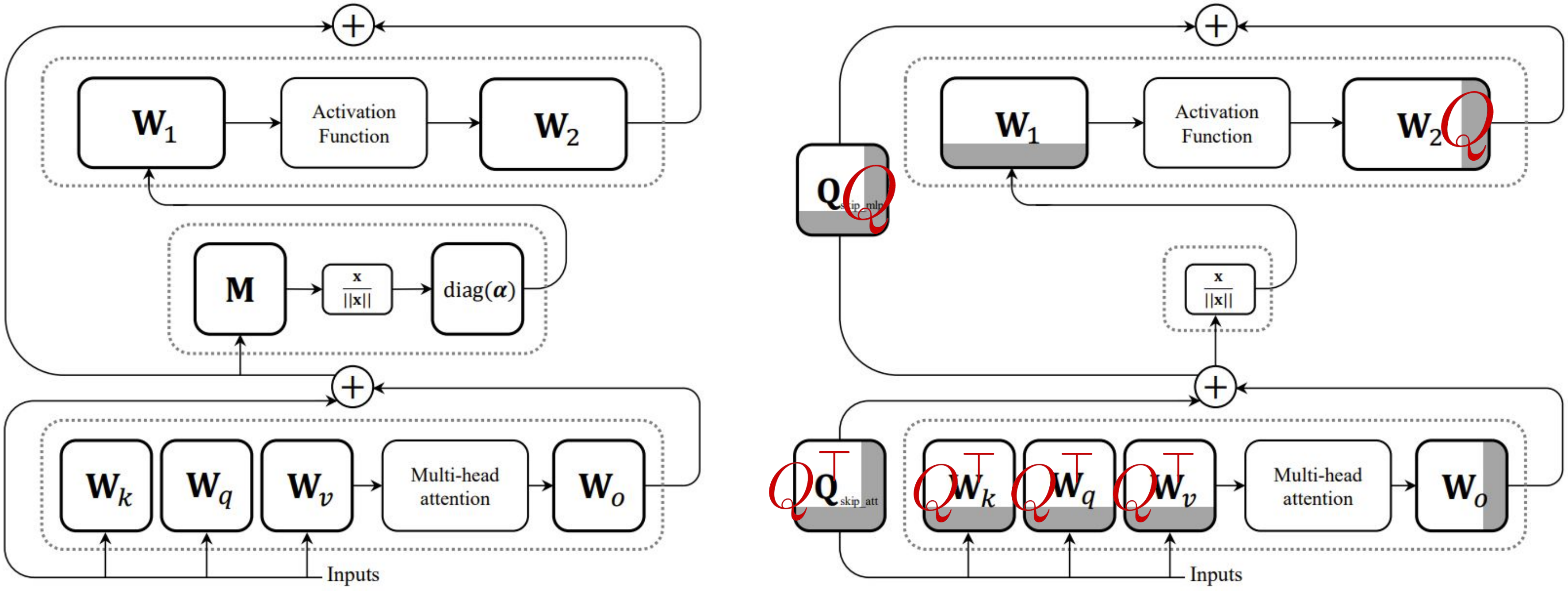
SliceGPT introduces rotational symmetries:



(a) A standard transformer block.

(b) A transformer block with SliceGPT.

SliceGPT introduces rotational symmetries:



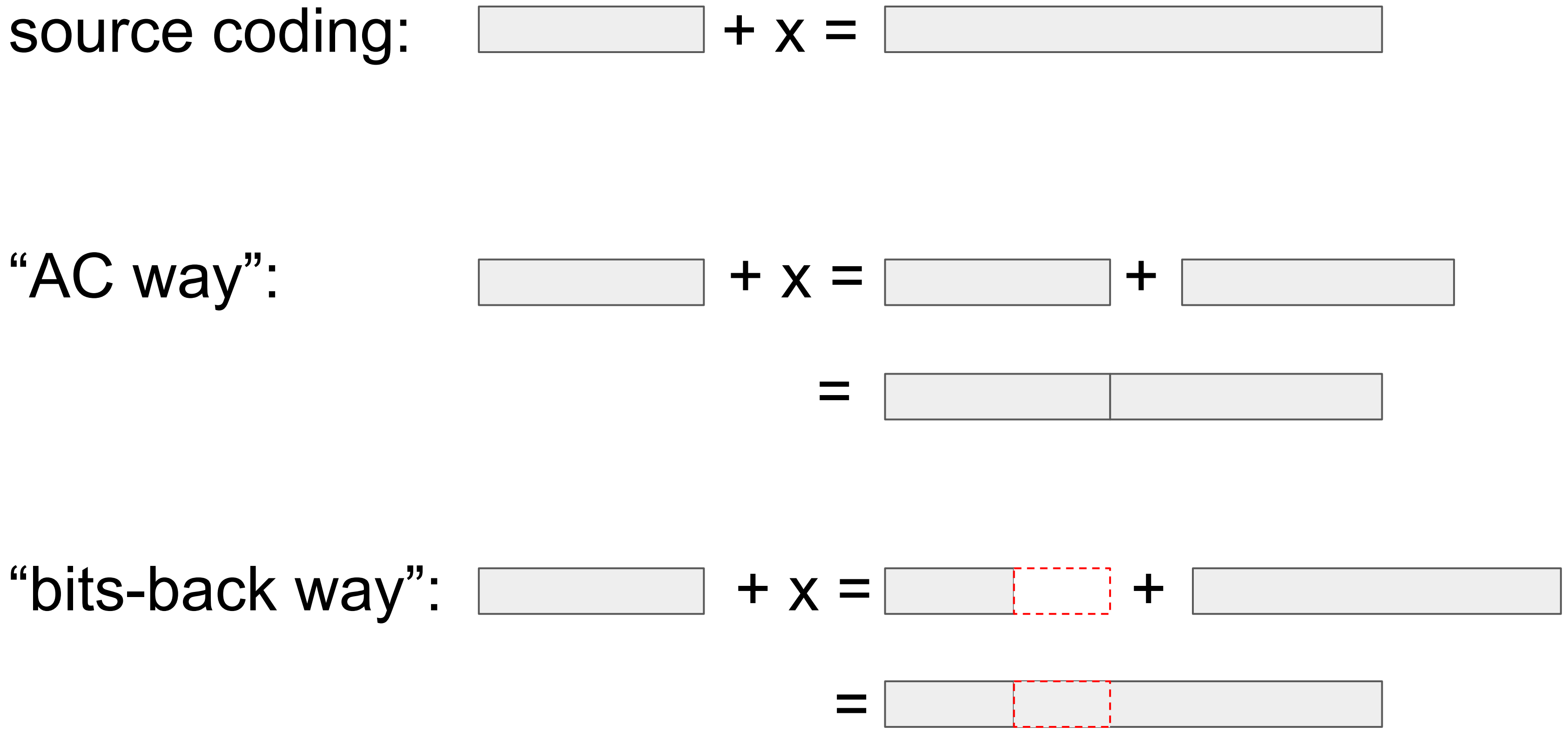
(a) A standard transformer block.

(b) A transformer block with SliceGPT.

SliceGPT introduces rotational symmetries:

we can write each transformer block as:

$$f(x, W) = f(x, QW)$$

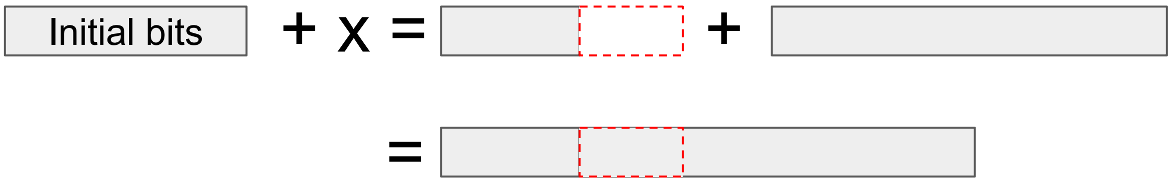

source coding:
+ x =
“AC way”:
+ x =
+
=
“bits-back way”:
+ x =
+
=

x~P, symmetric P + good code for P

How to compress |x|, x~P?

1. Remove the last bit of the initial bitstream.
2. If it’s 0, encode |x|; if 1, encode -|x|.

“bits-back way”:



Getting free bits back from rotational symmetries

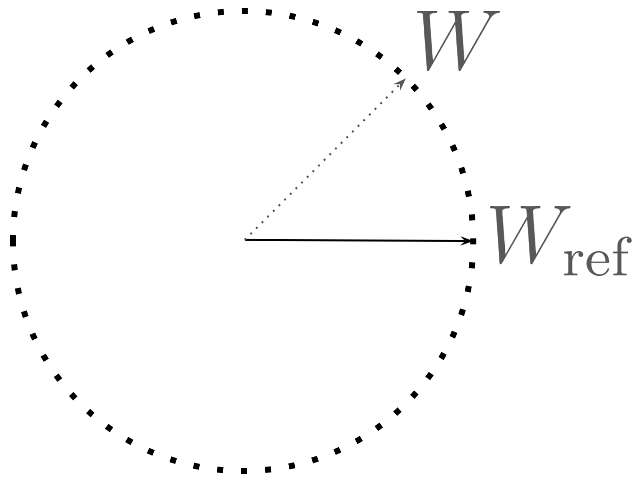
$$f(x, W) = f(x, QW)$$

encode:

encode: start with a weight matrix and some initial bits

encode step 1: rotate weight matrix to a “canonical” direction

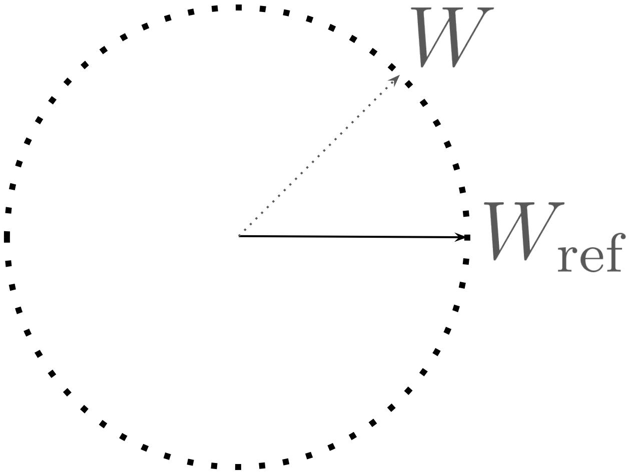
Initial bits



encode step 1: rotate weight matrix to a "canonical" direction

$$\mathrm{svd}(W) = U \underbrace{\Sigma V^\top}_{W_{\mathrm{ref}}}$$

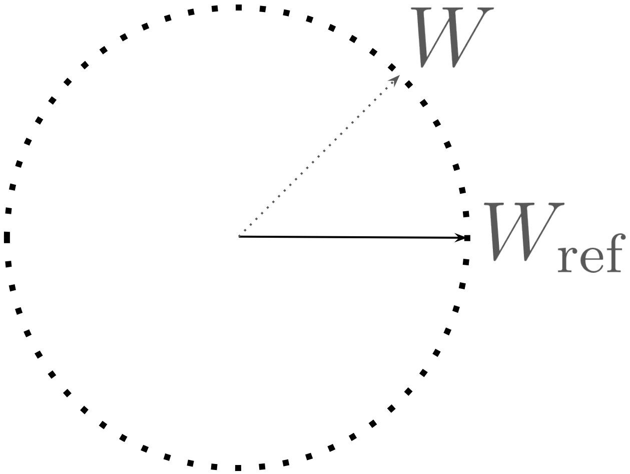
Initial bits



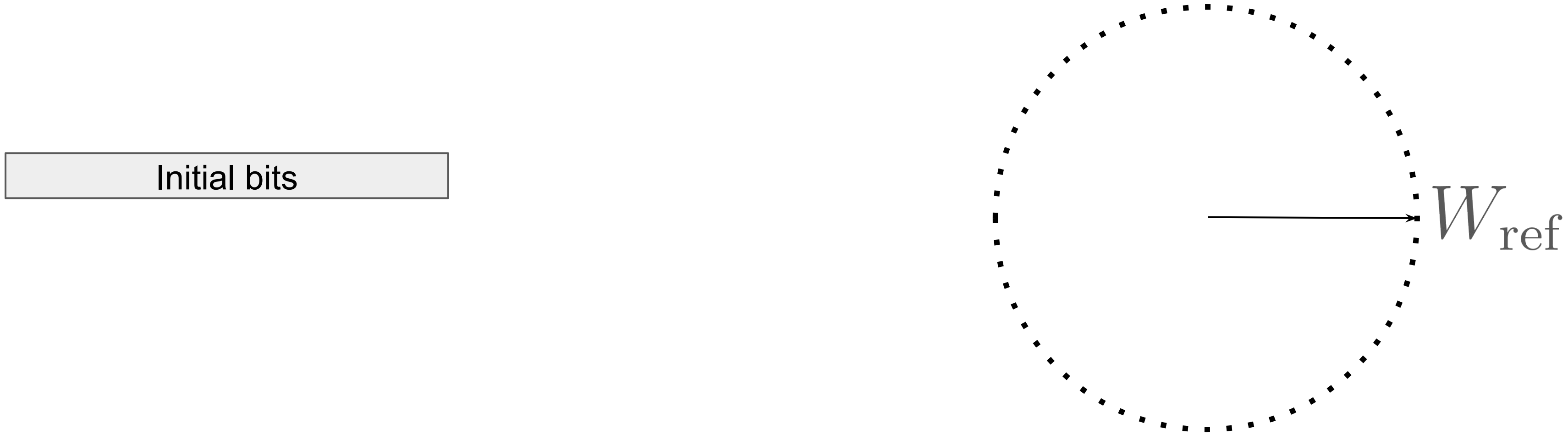
encode step 1: rotate weight matrix to a “canonical” direction

$$\mathrm{svd}(W) = U\underbrace{\Sigma V^\top}_{W_{\mathrm{ref}}}$$

i.e., define $W_{\mathrm{ref}} W_{\mathrm{ref}}^\top$ to be diagnoal

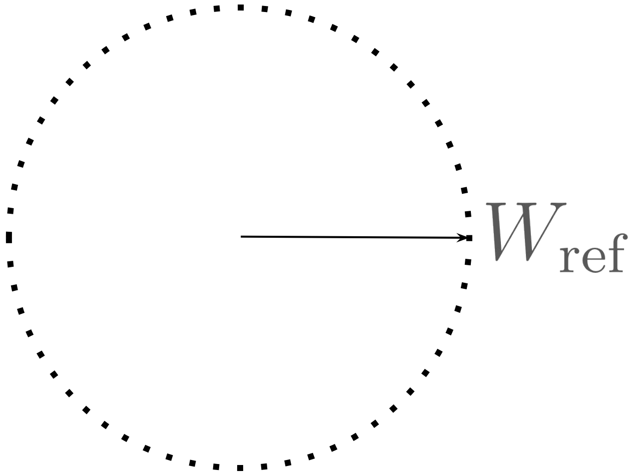


encode step 2: decode a rotation from the bitstream



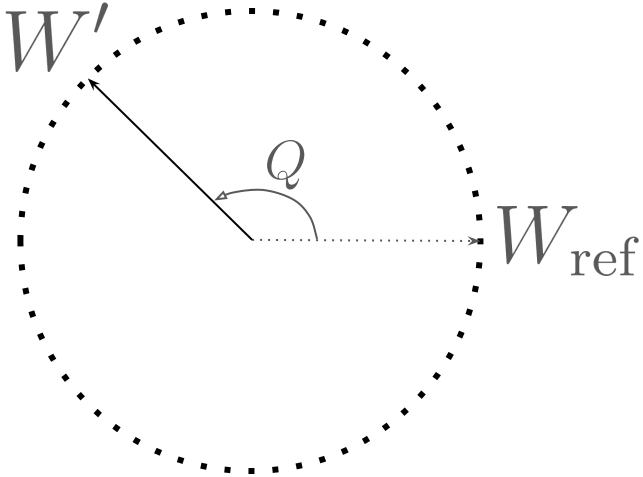
encode step 2: decode a rotation from the bitstream

encode step 3: rotate weight by the decoded rotation matrix

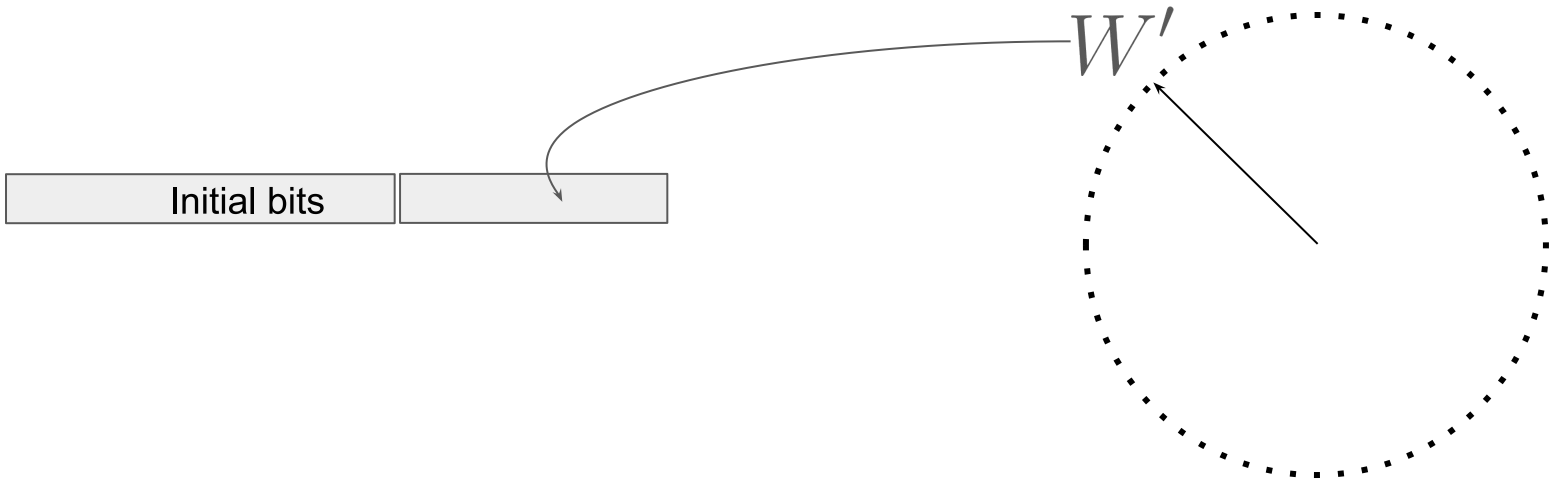


encode step 3: rotate weight by the decoded rotation matrix

$$W' = QW_{\text{ref}}$$

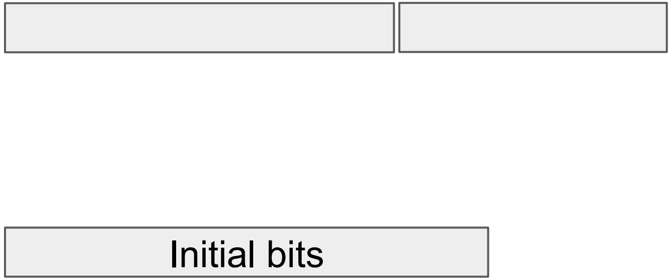


encode step 4: encode the rotated weight matrix

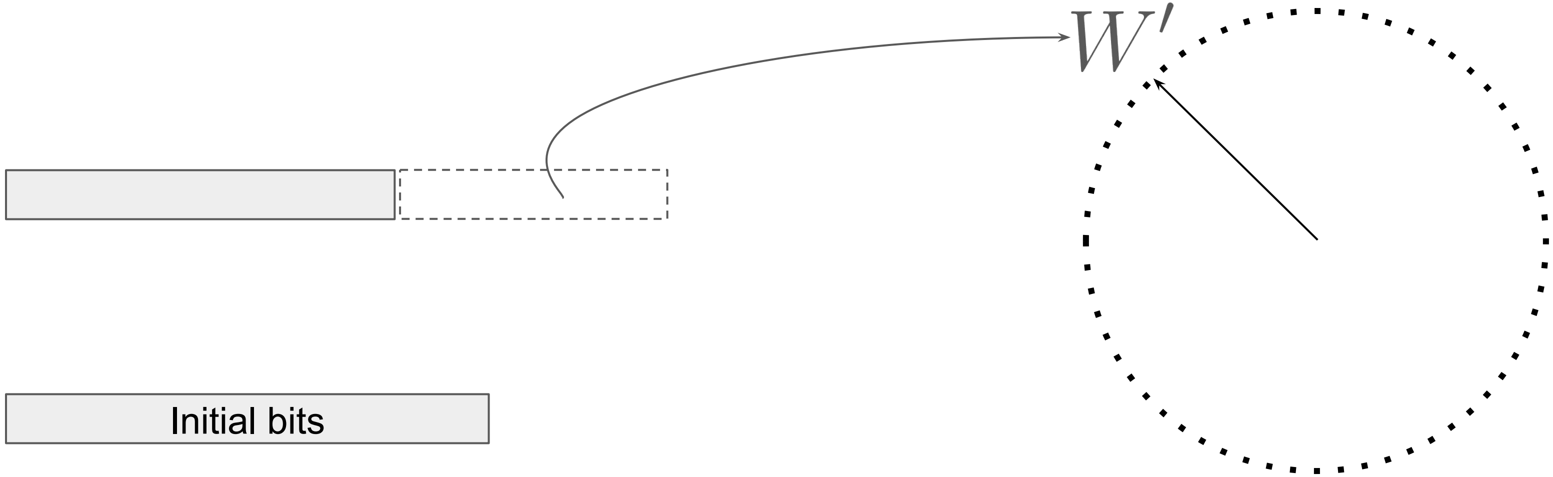


decode:

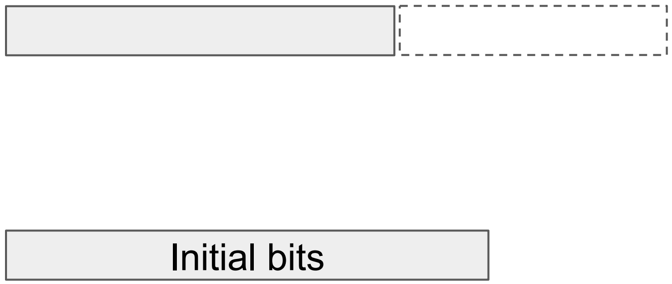
decode: start with some bits

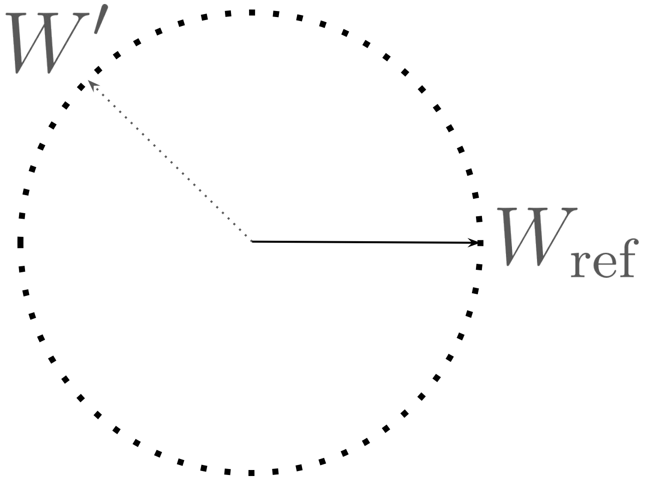


decode step 1: decode the rotated weight matrix



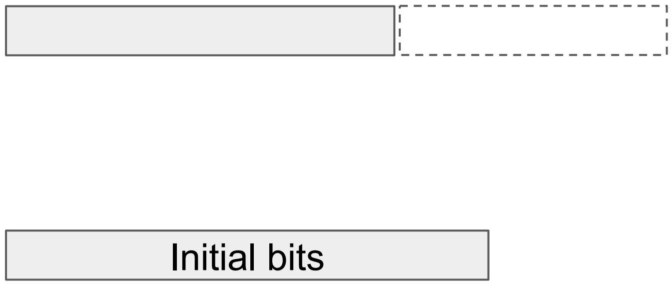
decode step 2: rotate it to “canonical” direction

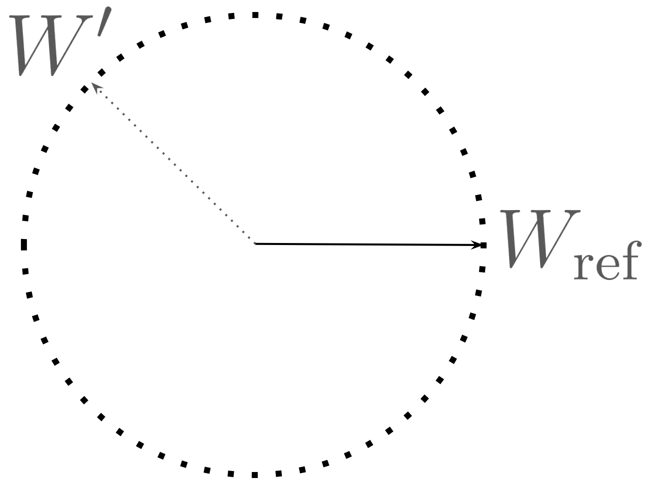




decode step 2: rotate it to “canonical” direction

recall we define $\mathrm{svd}(W) = U\underbrace{\Sigma V^\top}_{W_{\mathrm{ref}}}$

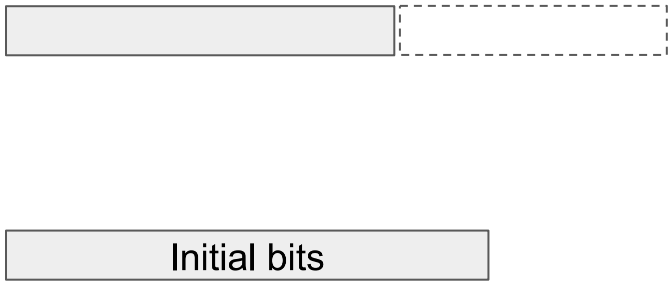


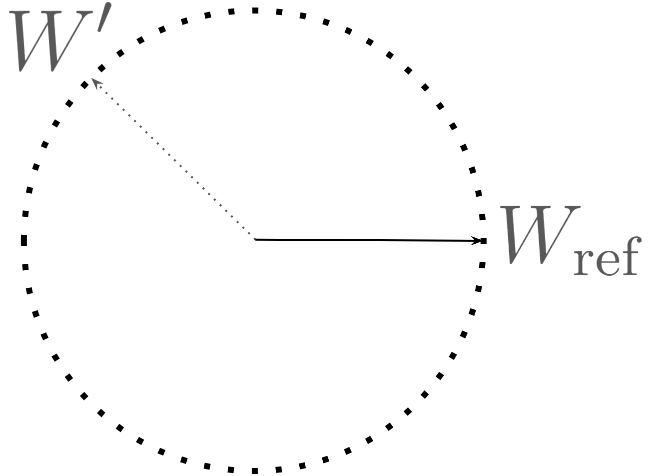


decode step 2: rotate it to “canonical” direction

recall we define $\mathrm{svd}(W) = U\underbrace{\Sigma V^\top}_{W_{\mathrm{ref}}}$

$$\mathrm{svd}(W') = \underbrace{\sigma Q}_{\hat{Q}}\Sigma V^\top\sigma,\ \sigma = \mathrm{diag}(\pm 1, \cdots, \pm 1)$$

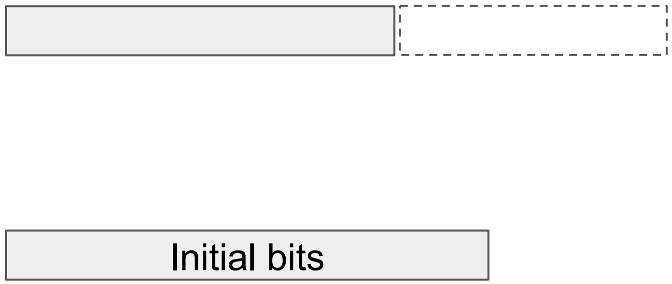


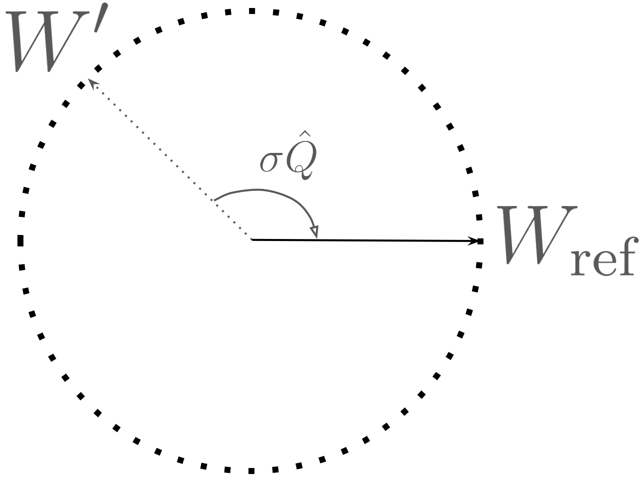


decode step 2: rotate it to “canonical” direction

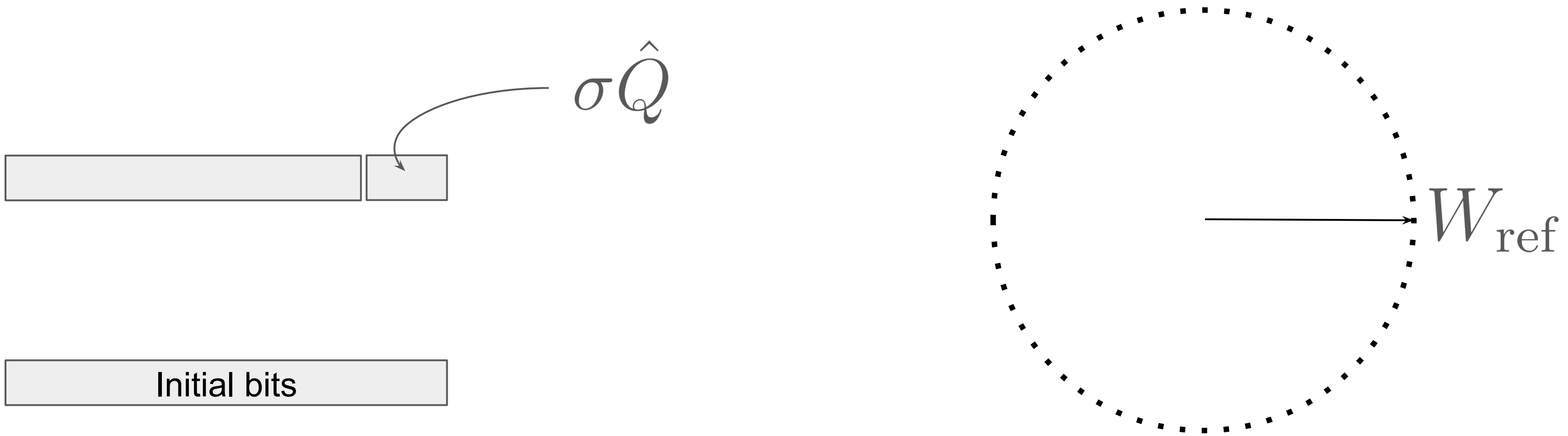
recall we define $\mathrm{svd}(W) = U\underbrace{\Sigma V^\top}_{W_{\mathrm{ref}}}$

$$\mathrm{svd}(W') = \underbrace{\sigma Q}_{\hat{Q}}\Sigma V^\top\sigma,\ \sigma = \mathrm{diag}(\pm 1, \cdots, \pm 1)$$





decode step 3: encode the recovered rotation matrix



encoding:

1. rotation to canonical direction
2. decode a rotation
3. rotate weight matrix
4. encode the rotated matrix

decoding:

3. encode the rotation

2. rotate weight to canonical direction

1. decode the rotated matrix

encoding:

1. rotation to canonical direction
2. decode a rotation
3. rotate weight matrix
4. encode the rotated matrix

decoding:

3. encode the rotation

2. rotate weight to canonical direction

1. decode the rotated matrix

😟 **does rotation need infinite precision?**

encoding:	**decoding:**
1. rotation to canonical direction	
2. decode a rotation	3. encode the rotation
3. rotate weight matrix	2. rotate weight to canonical direction
4. encode the rotated matrix	1. decode the rotated matrix

😟 **does rotation need infinite precision?**

😊 **yes. but just using float16 also works well!**

encoding:

1. rotation to canonical direction
2. decode a rotation
3. rotate weight matrix
4. encode the rotated matrix

decoding:

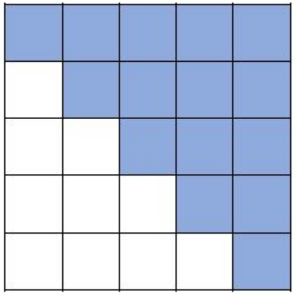
1. decode the rotated matrix
2. rotate weight to canonical direction
3. encode the rotation

does rotation need infinite precision?

yes. but just using float16 also works well!

encoding:

decoding:



canonical direction

otation

Decode $D(D+1)/2$ float

ght matrix

4. encode the rotated matrix

3. encode the rotation

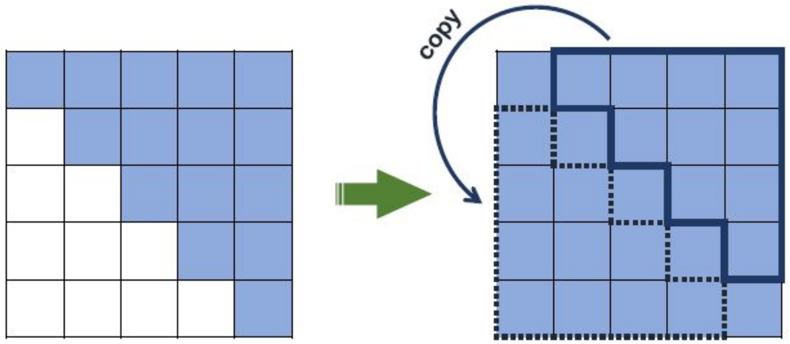
2. rotate weight to canonical direction

1. decode the rotated matrix

😟 **does rotation need infinite precision?**

😊 **yes. but just using float16 also works well!**

encoding:



Decode $D(D+1)/2$ float

Form a symmetric matrix

4. encode the rotated matrix

decoding:

3. encode the rotation

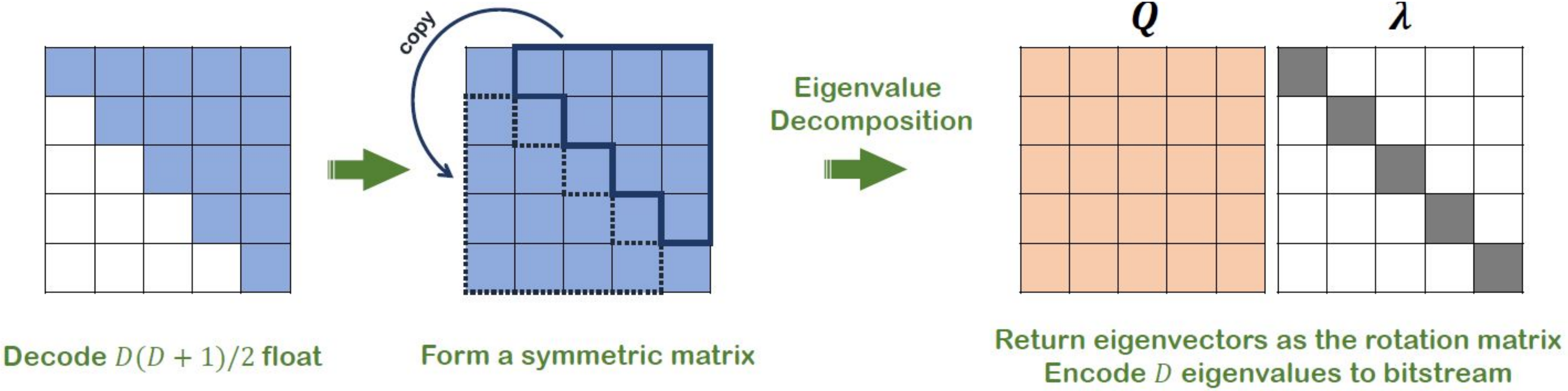
2. rotate weight to canonical direction

1. decode the rotated matrix

😟 **does rotation need infinite precision?**

😊 **yes. but just using float16 also works well!**

encoding: decoding:



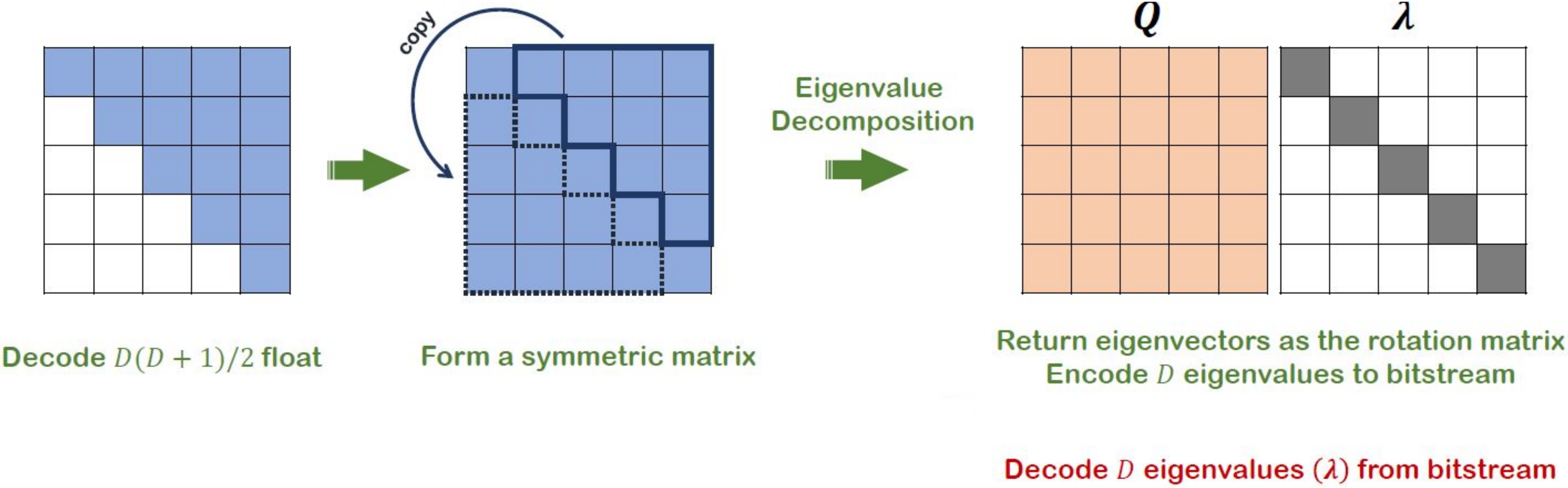
4. encode the rotated matrix 1. decode the rotated matrix

😟 **does rotation need infinite precision?**

😊 **yes. but just using float16 also works well!**

encoding:

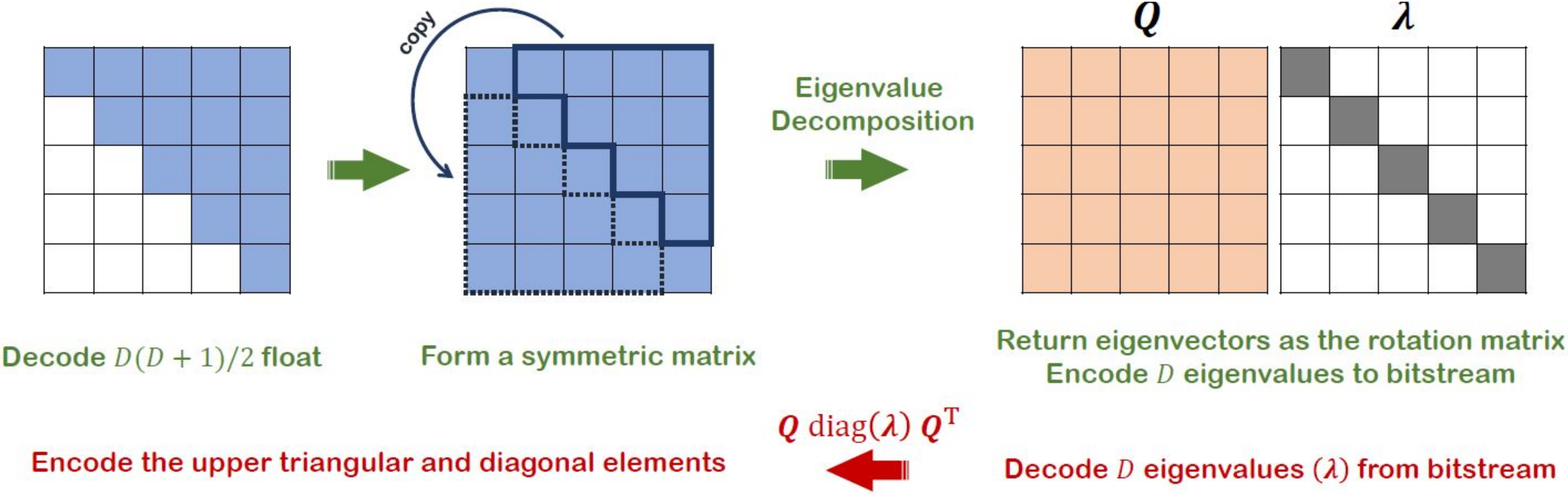
decoding:



😟 does rotation need infinite precision?

😊 yes. but just using float16 also works well!

encoding: decoding:



😟 **does rotation need infinite precision?**

😊 **yes. but just using float16 also works well!**

encoding:

1. rotation to canonical direction
2. decode a rotation
3. rotate weight matrix
4. encode the rotated matrix

decoding:

3. encode the rotation
2. rotate weight to canonical direction
1. decode the rotated matrix

😟 **but there may be numerical error…**

encoding:

1. rotation to canonical direction
2. decode a rotation
3. rotate weight matrix
4. encode the rotated matrix

≠

decoding:

3. encode the rotation
2. rotate weight to canonical direction
1. decode the rotated matrix

😟 **but there may be numerical error…**

encoding:

1. rotation to canonical direction
2. decode a rotation
3. rotate weight matrix
4. encode the rotated matrix

≠

decoding:

3. encode the rotation
2. rotate weight to canonical direction
1. decode the rotated matrix

😟 **but there may be numerical error…**

😟 **but there may be numerical error…**

💡 **We can send correction code if the error is too large!**

😟 **but there may be numerical error…**

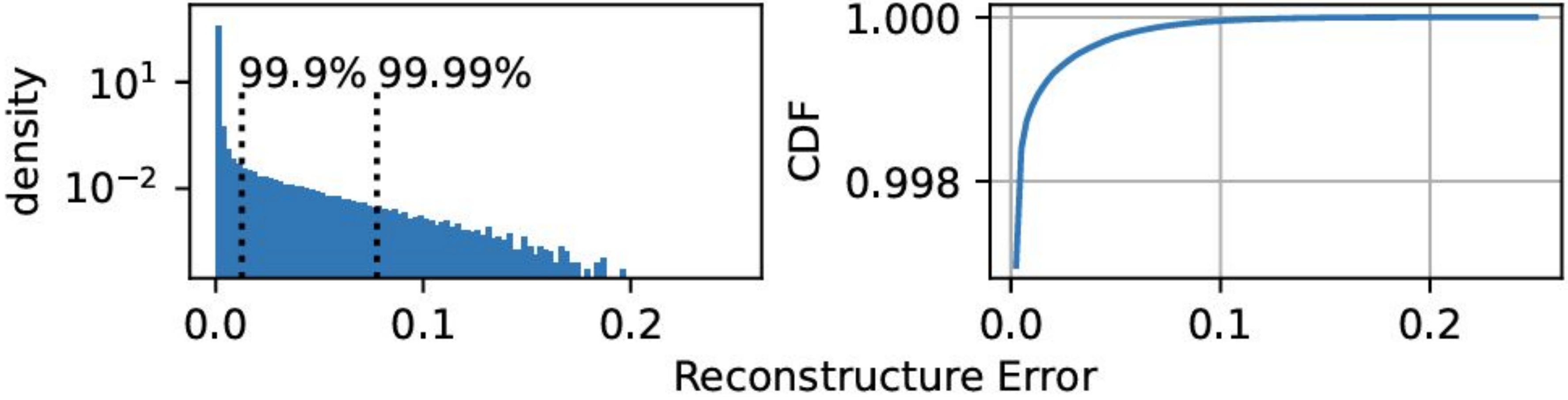
💡 **We can send correction code if the error is too large!**

Will this correction code becomes too large? NO!

😟 **but there may be numerical error…**

💡 **We can send correction code if the error is too large!**

Will this correction code becomes too large? NO!



Results

Results

Model	SliceGPT Slicing	Compress Rate after SliceGPT	Compress Rate after bits-back	Performance (before/after bits-back)			
				PPL (↓)	PIQA (%, ↑)	WinoGrande (%, ↑)	HellaSwag (%, ↑)
OPT-1.3B	20%	-9.53%	-13.77%	**16.59**/16.60	**64.91**/64.80	**54.78**/54.38	45.26/**45.32**
	25%	-14.84%	-18.61%	**17.78**/17.86	**63.55**/63.33	52.80/**53.28**	**43.20**/43.11
	30%	-20.53%	-23.81%	**19.60**/19.66	**60.88**/60.50	52.88/**53.28**	**40.25**/40.06
OPT-2.7B	20%	-9.19%	-13.84%	**13.89**/13.95	**68.44**/68.12	**58.88**/58.72	**51.35**/51.17
	25%	-15.07%	-19.09%	**14.85**/14.87	**66.70**/66.76	57.30/**57.70**	**48.41**/48.38
	30%	-20.88%	-24.43%	**16.31**/16.33	64.64/**64.69**	55.80/**56.04**	44.52/**44.57**
OPT-6.7B	20%	-9.29%	-14.07%	**11.63**/11.71	72.91/**73.01**	**61.33**/61.17	60.53/**60.55**
	25%	-15.16%	-19.29%	**12.12**/12.15	71.00/**71.22**	60.30/**60.77**	**57.76**/57.55
	30%	-21.18%	-24.84%	**12.81**/12.91	69.31/**69.42**	**59.75**/59.59	**53.64**/52.94
OPT-13B	20%	-9.18%	-14.01%	**10.75**/10.77	74.27/74.27	**64.96**/64.88	65.74/**65.79**
	25%	-15.27%	-19.51%	11.08/**11.07**	**74.27**/73.72	63.46/**63.93**	**63.48**/63.09
	30%	-21.29%	-24.97%	**11.55**/11.59	72.69/**73.01**	61.96/**62.43**	**60.12**/60.05
Llama-2-7B	20%	-9.38%	-14.13%	**6.86**/6.98	**69.53**/69.42	64.17/**64.72**	**58.96**/58.89
	25%	-15.34%	-19.53%	**7.56**/7.59	67.03/**67.57**	62.98/**63.38**	**54.29**/53.93
	30%	-21.45%	-25.09%	**8.63**/8.69	**64.69**/64.09	**62.75**/62.12	**49.13**/49.07

Results

3-5% additional bits saving

Model	SliceGPT Slicing	Compress Rate after SliceGPT	Compress Rate after bits-back	Performance (before/after bits-back)			
				PPL (↓)	PIQA (%, ↑)	WinoGrande (%, ↑)	HellaSwag (%, ↑)
OPT-1.3B	20%	-9.53%	-13.77%	**16.59**/16.60	**64.91**/64.80	**54.78**/54.38	45.26/**45.32**
	25%	-14.84%	-18.61%	**17.78**/17.86	**63.55**/63.33	52.80/**53.28**	**43.20**/43.11
	30%	-20.53%	-23.81%	**19.60**/19.66	**60.88**/60.50	52.88/**53.28**	**40.25**/40.06
OPT-2.7B	20%	-9.19%	-13.84%	**13.89**/13.95	**68.44**/68.12	**58.88**/58.72	**51.35**/51.17
	25%	-15.07%	-19.09%	**14.85**/14.87	**66.70**/66.76	57.30/**57.70**	**48.41**/48.38
	30%	-20.88%	-24.43%	**16.31**/16.33	64.64/**64.69**	55.80/**56.04**	44.52/**44.57**
OPT-6.7B	20%	-9.29%	-14.07%	**11.63**/11.71	72.91/**73.01**	**61.33**/61.17	60.53/**60.55**
	25%	-15.16%	-19.29%	**12.12**/12.15	71.00/**71.22**	60.30/**60.77**	**57.76**/57.55
	30%	-21.18%	-24.84%	**12.81**/12.91	69.31/**69.42**	**59.75**/59.59	**53.64**/52.94
OPT-13B	20%	-9.18%	-14.01%	**10.75**/10.77	74.27/74.27	**64.96**/64.88	65.74/**65.79**
	25%	-15.27%	-19.51%	11.08/**11.07**	**74.27**/73.72	63.46/**63.93**	**63.48**/63.09
	30%	-21.29%	-24.97%	**11.55**/11.59	72.69/**73.01**	61.96/**62.43**	**60.12**/60.05
Llama-2-7B	20%	-9.38%	-14.13%	**6.86**/6.98	**69.53**/69.42	64.17/**64.72**	**58.96**/58.89
	25%	-15.34%	-19.53%	**7.56**/7.59	67.03/**67.57**	62.98/**63.38**	**54.29**/53.93
	30%	-21.45%	-25.09%	**8.63**/8.69	**64.69**/64.09	**62.75**/62.12	**49.13**/49.07

Results

Model	SliceGPT Slicing	Compress Rate after SliceGPT	Compress Rate after bits-back	Performance (before/after bits-back)			
				PPL (↓)	PIQA (%, ↑)	WinoGrande (%, ↑)	HellaSwag (%, ↑)
OPT-1.3B	20%	-9.53%	-13.77%	**16.59**/16.60	**64.91**/64.80	**54.78**/54.38	45.26/**45.32**
	25%	-14.84%	-18.61%	**17.78**/17.86	**63.55**/63.33	52.80/**53.28**	**43.20**/43.11
	30%	-20.53%	-23.81%	**19.60**/19.66	**60.88**/60.50	52.88/**53.28**	**40.25**/40.06
OPT-2.7B	20%	-9.19%	-13.84%	**13.89**/13.95	**68.44**/68.12	**58.88**/58.72	**51.35**/51.17
	25%	-15.07%	-19.09%	**14.85**/14.87	**66.70**/66.76	57.30/**57.70**	**48.41**/48.38
	30%	-20.88%	-24.43%	**16.31**/16.33	64.64/**64.69**	55.80/**56.04**	44.52/**44.57**
OPT-6.7B	20%	-9.29%	-14.07%	**11.63**/11.71	72.91/**73.01**	**61.33**/61.17	60.53/**60.55**
	25%	-15.16%	-19.29%	**12.12**/12.15	71.00/**71.22**	60.30/**60.77**	**57.76**/57.55
	30%	-21.18%	-24.84%	**12.81**/12.91	69.31/**69.42**	**59.75**/59.59	**53.64**/52.94
OPT-13B	20%	-9.18%	-14.01%	**10.75**/10.77	74.27/74.27	**64.96**/64.88	65.74/**65.79**
	25%	-15.27%	-19.51%	11.08/**11.07**	**74.27**/73.72	63.46/**63.93**	**63.48**/63.09
	30%	-21.29%	-24.97%	**11.55**/11.59	72.69/**73.01**	61.96/**62.43**	**60.12**/60.05
Llama-2-7B	20%	-9.38%	-14.13%	**6.86**/6.98	**69.53**/69.42	64.17/**64.72**	**58.96**/58.89
	25%	-15.34%	-19.53%	**7.56**/7.59	67.03/**67.57**	62.98/**63.38**	**54.29**/53.93
	30%	-21.45%	-25.09%	**8.63**/8.69	**64.69**/64.09	**62.75**/62.12	**49.13**/49.07

negligible influence on performance

Encoding and Decoding time

GPU:

Model Name	OPT-1.3B		OPT-2.7B		OPT-6.7B		OPT-13B	
Slicing	20%	30%	20%	30%	20%	30%	20%	30%
Encoding time	15 s	13 s	30 s	24 s	2.5 min	1.7 min	6.5 min	4.1 min
Decoding time	6 s	5 s	14 s	11 s	1.2 min	45 s	2.5 min	2 min

CPU:

Model Name	OPT-1.3B		OPT-2.7B		OPT-6.7B		OPT-13B	
Slicing	20%	30%	20%	30%	20%	30%	20%	30%
Encoding time	3.9 min	3.5 min	8 min	6.5 min	30 min	25 min	84 min	68 min
Decoding time	1.5 min	1.5 min	3.5 min	2.5 min	12 min	10 min	30 min	24 min

In summary:

save 3-5% additional bits,

In summary:

save 3-5% additional bits,
no influence on performance,

In summary:

save 3-5% additional bits,

no influence on performance,

a little overhead in model loading/saving time